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# Vibrational modes in the dust-plasma crystal

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It is shown that vertical vibration of dust grains in a sheath region can lead to a specific low frequency mode of a quasi-two-dimensional dust-plasma crystal. Linear dispersion characteristics of the mode are obtained. [S1063-651X(97)51607-2]

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Since the possibility of formation of Coulomb quasilattices [1] involving micrometer sized highly charged dust particulates has been demonstrated experimentally [2–6], there is growing world-wide theoretical and experimental effort to investigate these strongly coupled systems. The dust-crystal structure is observed in the sheath region where there is balance between the gravitational and electrostatic forces, and in most of the experiments it consists of just a few layers of dust particles levitating above the horizontal negatively biased electrode [4,6–11].

Recently, it has been demonstrated [12] that lattice waves can propagate in the one-dimensional chain of strongly coupled dust particles. However, the waves theoretically studied in [12] included motion of dust particles only in the horizontal direction. At the same time, motion of the dust grains in the vertical direction can provide a useful tool for determining the grain charge [6,8–10].

In this Rapid Communication, we demonstrate that oscillations of dust grains in the vertical plane can lead to a low-frequency mode. We note that excitation of this mode can also be responsible for phase transitions in the system, which are the subject of recent interest, see, e.g., [7,8]. The mode is characterized by an optic-mode-like inverse dispersion (i.e., its frequency decreases with the growing wave number) if $kr_0 \ll 1$, where $k$ is the wave number, $r_0$ is the interparticle distance, and only nearest-neighbor interactions are taken into account.

Consider vibrations of a one-dimensional horizontal chain of particulates of equal mass $M$ separated by the distance $r_0$; see Fig. 1. We assume that the interaction potential between particles can be approximated by the Debye law

$$\Phi = \frac{Q}{r}\exp\left(-\frac{r}{\lambda_D}\right), \qquad (1)$$

where $\lambda_D$ is the screening length of the dust grain charge $Q$ by plasma particles. In addition to the electrostatic Debye shielded force, the gravitational force $Mg$ and the sheath electrostatic force $F_e = QE(z)$ act on the dust grains in the vertical direction $z$. The balance of forces in the linear approximation with respect to small perturbations $\delta z$ of the equilibrium at $z=0$ gives the equation for vertical oscillations

$$M\frac{d^2\delta z_n}{dt^2} = \frac{Q^2}{r_0^3}\,e^{-r_0/\lambda_D}(1+r_0/\lambda_D)(2\,\delta z_n - \delta z_{n-1} - \delta z_{n+1}) - Mg + F_e . \qquad (2)$$

Here

$$F_e - Mg = -\gamma \delta z_n , \qquad (3)$$

where $\gamma$ is a constant assuming linear variation of the sheath electric field, and $\delta z_n$ is the vertical deviation of a particle number $n$ from its equilibrium position. We note that although in general particles oscillate in the vertical as well as in horizontal direction, see Fig. 1, in the linear approximation their transverse vibrations (which are the subject of this paper) and longitudinal vibrations are not coupled. Substituting $\delta z_n = A\exp(-i\omega t + iknr_o)$ into Eq. (2) gives the dispersion of the vertical oscillations

$$\omega^2 = \frac{\gamma}{M} - \frac{4Q^2}{Mr_0^3}\,e^{-r_0/\lambda_D}(1+r_0/\lambda_D)\sin^2\frac{kr_0}{2}. \qquad (4)$$

We see that for $k=0$ the characteristic frequency is given by $\omega^2 = \gamma/M$, decreasing with growing wave number when $kr_0 \ll 1$.

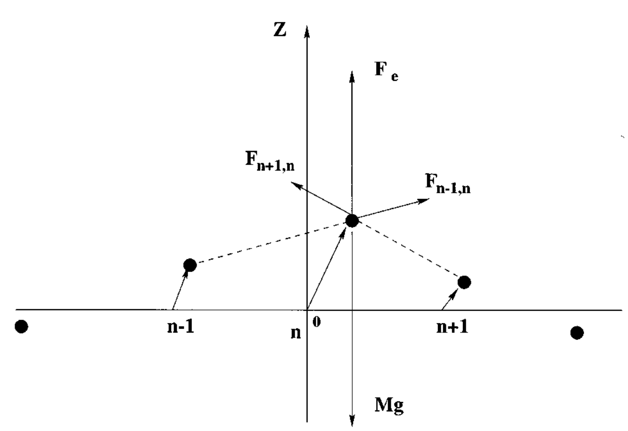


FIG. 1. Oscillations of dust particles in a one-dimensional chain.

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To estimate the effective width of the potential well $\gamma$, we consider the standard model of the sheath [13], which considers Boltzmann distributed electrons. For simplicity, we ignore the influence the dust grains may cause on the field distribution in the sheath region. The ion continuity equation gives the ion density $n_i$ in terms of the density $n_0$ in the plasma bulk

$$n_i(z)=n_0\left[1-\frac{2e\phi(z)}{m_i v_0^2}\right]^{-1/2}, \tag{5}$$

where $v_0$ is the speed of the ion flow towards the negatively charged electrode, $m_i$ is the ion mass, and $\phi(z)$ is the sheath potential. From Poisson's equation, we then obtain

$$\frac{d^2\phi(z)}{dz^2}=4\pi e n_0\left[\exp\left(\frac{e\phi(z)}{T_e}\right)-\left(1-\frac{2e\phi(z)}{m_i v_0^2}\right)^{-1/2}\right]. \tag{6}$$

This equation can be integrated once to give [applying the boundary conditions $E(\infty)=\phi(\infty)=0$]

$$E^2(z)=8\pi n_0 T_e\left\{\exp\left(\frac{e\phi(z)}{T_e}\right)-1+\frac{v_0^2}{v_s^2}\left[\left(1-\frac{2e\phi(z)}{T_e}\frac{v_s^2}{v_0^2}\right)^{1/2}-1\right]\right\}, \tag{7}$$

where $v_s^2=T_e/m_i$ is the squared sound speed.

Linearizing Eq. (7) with respect to small potential and field variations, we find

$$\delta E\approx\frac{4\pi n_0 T_e}{E_0}\left[\exp\left(\frac{e\phi_0}{T_e}\right)-\left(1-\frac{2e\phi_0}{T_e}\frac{v_s^2}{v_0^2}\right)^{-1/2}\right]\frac{e\delta\phi}{T_e}, \tag{8}$$

where the electric field is $E_0$ at $z=0$, and for the dust grains it is balanced by the gravity

$$|QE_0|=Mg. \tag{9}$$

Now, we assume that the sheath electric field near the position of the dust grains can be linearly approximated. Thus we find

$$\delta E\approx-4\pi n_0 e\left[\exp\left(\frac{e\phi_0}{T_e}\right)-\left(1-\frac{2e\phi_0}{T_e}\frac{v_s^2}{v_0^2}\right)^{-1/2}\right]\delta z, \tag{10}$$

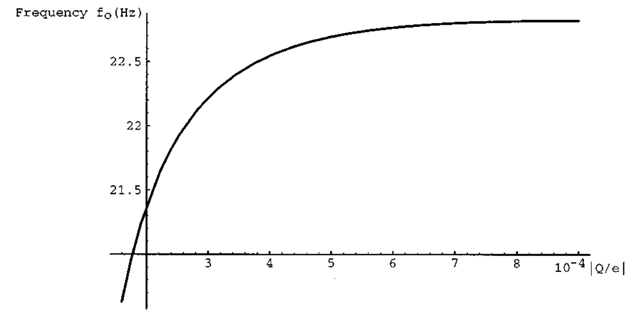


FIG. 2. Frequency $f_0$ in Hz vs dust charge $10^{-4}|Q/e|$.

and the effective width of the potential well is given by

$$\gamma=4\pi e|Q|n_0\left[\exp\left(\frac{e\phi_0}{T_e}\right)-\left(1-\frac{2e\phi_0}{T_e}\frac{v_s^2}{v_0^2}\right)^{-1/2}\right]. \tag{11}$$

Equations (7) and (9) for the sheath potential $\phi_0$ in the equilibrium position can be solved numerically. When $e\phi_0 \ll T_e$, the characteristic frequency is approximately given by

$$f_0=\frac{1}{2\pi}\sqrt{\frac{\gamma}{M}}\approx\frac{1}{2\pi}\sqrt{\frac{g(1-v_s^2/v_0^2)}{\lambda_D}}\simeq 20Hz, \tag{12}$$

where we assumed $\lambda_D\approx(T_e/4\pi n_0 e^2)^{1/2}\sim 2\times10^{-2}$ cm and $v_0^2/v_s^2\sim1.5$. Figure 2 presents numerical result for the frequency $f_0$ as a function of the dust charge $Q$.

To conclude, we have demonstrated that the vertical oscillations of a one-dimensional chain of dust grains levitating in the sheath field of a horizontal negatively biased electrode can give rise to a specific low-frequency mode that is characterized by inverse optic-mode-like dispersion when the wavelengths far exceed the intergrain distance. Excitation of the mode may stimulate phase transitions in the system. We note that considering the one-dimensional chain we ignored the ion drag force, which is not important in this situation. However, for several layers of dust grains in the vertical direction, the ion drag force, which in a plasma with finite flows necessarily includes collective effects [14–16], must be taken into account. This is a subject for further study.

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